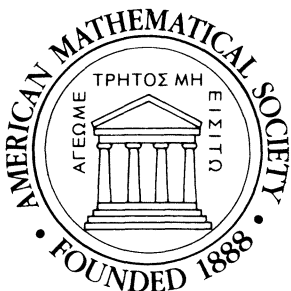


# Mathematics of Computation



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July 1989

Volume 53, Number 187, Pages 1–454

**Published by the American Mathematical Society  
Providence, Rhode Island USA**

ISSN 0025-5718

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This publication was typeset using AMS-TEX,  
the American Mathematical Society's TEX macro system.

# MATHEMATICS OF COMPUTATION

## TABLE OF CONTENTS

July 1989

<b>James H. Bramble, Joseph E. Pasciak, and Alfred H. Schatz</b> , The Construction of Preconditioners for Elliptic Problems by Substructuring, IV . . . . .	1
<b>M. Crouzeix, V. Thomée, and L. B. Wahlbin</b> , Error Estimates for Spatially Discrete Approximations of Semilinear Parabolic Equations with Initial Data of Low Regularity . . . . .	25
<b>T. Geveci</b> , On the Convergence of a Time Discretization Scheme for the Navier-Stokes Equations . . . . .	43
<b>Michelle Schatzman and Michel Bercovier</b> , Numerical Approximation of a Wave Equation with Unilateral Constraints . . . . .	55
<b>W. H. Hundsdorfer and J. G. Verwer</b> , Stability and Convergence of the Peaceman-Rachford ADI Method for Initial-Boundary Value Problems . . . . .	81
<b>Wilhelm Heinrichs</b> , Improved Condition Number for Spectral Methods . . . . .	103
<b>Vidar Thomée and Nai-Ying Zhang</b> , Error Estimates for Semidiscrete Finite Element Methods for Parabolic Integro-Differential Equations . . . . .	121
<b>Christoph Börgers</b> , On the Numerical Solution of the Regularized Birkhoff Equations . . . . .	141
<b>P. P. B. Eggermont</b> , Uniform Error Estimates of Galerkin Methods for Monotone Abel-Volterra Integral Equations on the Half-Line . . . . .	157
<b>George Marsaglia, Arif Zaman, and John C. W. Marsaglia</b> , Numerical Solution of Some Classical Differential-Difference Equations . . . . .	191
<b>D. J. Evans and C. Li</b> , Determination of the $D^{1/2}$ -Norm of the SOR Iterative Matrix for the Unsymmetric Case . . . . .	203
<b>Charles K. Chui and Tian Xiao He</b> , On the Dimension of Bivariate Superspline Spaces . . . . .	219
<b>G. Meinardus, G. Nürnberger, M. Sommer, and H. Strauss</b> , Algorithms for Piecewise Polynomials and Splines with Free Knots . . . . .	235
<b>Ömer Egecioğlu and Çetin K. Koç</b> , A Fast Algorithm for Rational Interpolation Via Orthogonal Polynomials . . . . .	249
<b>Bernd Fischer and Lothar Reichel</b> , Newton Interpolation in Fejér and Chebyshev Points . . . . .	265
<b>P. Rabinowitz and D. S. Lubinsky</b> , Noninterpolatory Integration Rules for Cauchy Principal Value Integrals . . . . .	279
<b>Yuan Xu</b> , Weight Functions for Chebyshev Quadrature . . . . .	297
<b>F. Locher</b> , A Family of Generalized Jacobi Polynomials . . . . .	303
<b>J. M. Borwein and P. B. Borwein</b> , On the Mean Iteration $(a, b) \leftarrow ((a + 3b)/4, (\sqrt{ab} + b)/2)$ . . . . .	311
<b>B. C. Carlson</b> , A Table of Elliptic Integrals: Cubic Cases . . . . .	327
<b>J. F. Loiseau, J. P. Codaccioni, and R. Caboz</b> , Incomplete Hyperelliptic Integrals and Hypergeometric Series . . . . .	335

<b>Lothar Afferbach and Rainer Weilbächer</b> , The Exact Determination of Rectangle Discrepancy for Linear Congruential Pseudorandom Numbers . . .	343
<b>David W. Boyd</b> , Reciprocal Polynomials Having Small Measure. II . . . . .	355
<b>David J. Rusin</b> , The Cohomology of the Groups of Order 32 . . . . .	359
<b>J. Buchmann, M. Pohst, and J. v. Schmettow</b> , On the Computation of Unit Groups and Class Groups of Totally Real Quartic Fields . . . . .	387
<b>János Pintz, William L. Steiger, and Endre Szemerédi</b> , Infinite Sets of Primes with Fast Primality Tests and Quick Generation of Large Primes . . . . .	399
<b>Wayne L. McDaniel</b> , Some Pseudoprimes and Related Numbers Having Special Forms . . . . .	407
<b>H. Dubner</b> , A New Method for Producing Large Carmichael Numbers . . . . .	411
<b>Ian Miyamoto and M. Ram Murty</b> , Elliptic Pseudoprimes . . . . .	415
<b>Richard P. Brent and Graeme L. Cohen</b> , A New Lower Bound for Odd Perfect Numbers . . . . .	431
<b>Landon Curt Noll and David I. Bell</b> , $n$ -clusters for $1 < n < 7$ . . . . .	439
<b>Reviews and Descriptions of Tables and Books</b> . . . . .	445
Tikhonov and Goncharsky, Editors <b>13</b> , Coleman and Van Loan <b>14</b> , Forgó <b>15</b> , Bullen, Mitrinović and Vasić <b>16</b> , Brass and Hämmerlin, Editors <b>17</b> , Wada <b>18</b> , Philippou, Horadam and Bergum, Editors <b>19</b>	
<b>Supplement to “Reciprocal Polynomials Having Small Measure. II”</b> by <b>David W. Boyd</b> . . . . .	S1
<b>Supplement to “A New Lower Bound for Odd Perfect Numbers”</b> by <b>Richard P. Brent and Graeme L. Cohen</b> . . . . .	S7

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